Logarithm, often called 'logs,' is the power to which a number must be raised to get the result. It is thus the inverse of the exponent and is written as: ba = x ⇔ logbx = a Here, 'b' is the base 'a' is the exponent 'x' is the argument are the parts of a logarithm. Thus, the logarithm represents the exponent to which a base is raised to yield a given number. For example, we know 43 = 64 Here, using the logarithm, we can answer how many 4s we multiply to get 64. Since 4 × 4 × 4 = 64, we multiply three 4s to get 64, which is written in the logarithmic form as log4(64) = 3, read as 'log base 4 of 64 is 3.' Thus, 43 = 64 ⇔ log4(64) = 3, where the base is 4, and the exponent or power is 3 Here are some examples of conversions from exponential to logarithmic form and vice-versa. Exponential FormLogarithmic Form54 = 625log5(625) = 4152 = 225log15(225) = 283 = 512log8(512) = 3 Find the value of log7(343).Solution:As we know, 7 × 7 × 7 = 73 = 343Thus, log7(343) = 3 Convert 35 = 243 in its logarithmic form.Solution:As we know, ba = x ⇔ logbx = aHere, 35 = 243⇒ log3(243) = 5, the required logarithmic form. However, the expression logbx has restrictions, which are as follows: The base 'b' of a logarithm is always a positive real number (b > 0) and does not equal 1 (b ≠ 1). For negative bases, logarithm leads to complex results. Now, let us assume the base is 1, and the equation is: log17 = x ⇒ 1x = 7 Since 1 raised to any power yields 1, 1x = 7 is false. Thus, the base does not equal 1. The argument 'x' is always a positive real number (a > 0). Since a positive number (b > 0) is raised to any power, it yields a positive number (bx > 0). Thus, ba = x follows that x > 0. The base of a logarithm can have many positive values except 1. However, two of them are frequently used. The logarithm whose base is 10 is known as the common logarithm or base-10-logarithm. It is often denoted as log(x) without a subscript. For example, log10(10000) = 4 ⇒ log(10000) = 4 ⇒ 104 = 10000 The logarithm with the base 'e' (= 2.718..., Euler's number) is the natural logarithm or base-e-logarithm, denoted by ln(x) or loge(x). For example, ln(e2) = 2 ⇒ e2 = e × e, ln(9) = c ⇒ ec = 9 Certain rules (also properties or identities) of logarithms are used to simplify, expand, or condense them. Let us simplify a logarithmic function log35 using the appropriate rule(s). Since logb(xy) = logbx + logby and 35 = 5 × 7 (Using Product rule) Thus, log35 = log(5 × 7) = log 5 + log 7 = 0.699 + 0.845 (rounded to 3 decimal places) ⇒ log35 = 1.544 (rounded to 3 decimal places) We observe that logarithms also have decimal values like 1.544, which means 101.544... = 35, and is graphically represented as: However, negative logarithms are formed when the argument is between 0 and 1. In logbx < 0, for 0 < x < 1, 'b' is the base, and 'x' is the argument. For example, log(0.0001) = -4 gives a negative value, and its exponential form, 10-4 = 0.0001, gives a decimal. Let us expand the logarithmic expression log(5x4y5). Here, log(5x4y5) Using the product rule, we get log(5) + log(x4) + log(y5) Using the power rule, we get log(5) + 4 log(x) + 5 log(y) Thus, log(5x4y5) = log(5) + 4 log(x) + 5 log(y) Considering the above example of log(5) + 4 log(x) + 5 log(y) and condensing it into a single logarithm, we get: log(5) + 4 log(x) + 5 log(y) Using the power rule, we get log(5) + log(x4) + log(y5) Using the product rule, we get log(5x4y5) Thus, log(5) + 4 log(x) + 5 log(y) = log(5x4y5) Evaluate ${\log \left(\dfrac{100x^{4}}{10y^{2}}\right) +2\log \left(y\right)}$Solution:Here, ${\log \left(\dfrac{100x^{4}}{10y^{2}}\right) +2\log \left(y\right)}$ = ${\log \left(\dfrac{10x^{4}}{y^{2}}\right) +2\log \left(y\right)}$Using the quotient rule, we get${\log \left(10x^{4}\right) -\log \left(y^{2}\right) +2\log \left(y\right)}$Using the product rule, we get${\log \left(10\right) +\log \left(x^{4}\right) -\log \left(y^{2}\right) +2\log \left(y\right)}$Using the power rule, we get${\log \left(10\right) +4\log \left(x\right) -2\log \left(y\right) +2\log \left(y\right)}$Since logbb = 1, log(10) = 1Thus, ${\log \left(\dfrac{100x^{4}}{10y^{2}}\right) +2\log \left(y\right)}$ = 1 + 4log(x) Expand the expression log2(16x8)Solution:Here, log2(16x8)Using the product law of logarithms, we getlog2(16) + log2(x8) = 4 + log2(x8)Using the power law of logarithms, we get4 + 8 log2(x)Thus, log2(16x8) = 4 + 8 log2(x) Condense the expression log5(9x2) – log5(x) + log5(8x4)Solution:Here, log5(9x2) – log5(x) + log5(8x4)First, we solve the quotient rule by subtracting the logarithms. The expression is: ${\log _{5}\left(\dfrac{9x^{2}}{x}\right) +\log _{5}\left(8x^{4}\right)}$= ${\log _{5}\left(9x\right) +\log _{5}\left(8x^{4}\right)}$Now, we use the product rule, and the expression becomes${\log _{5}\left(72x^{5}\right)}$Thus, log5(9x2) – log5(x) + log5(8x4) = ${\log _{5}\left(72x^{5}\right)}$ Solve the equation log(x) + log(3) = log(51) and find the value of x.Solution:Here, log(x) + log(3) = log(51)⇒ log(3x) = log(51) (by the product rule)Now, canceling the log from both sides, we get3x = 51⇒ x = 17Thus, x = 17 A nested logarithm, also called iterated logarithm, represents a logarithm within another logarithm and are expressed as: log(log(log...(logx))) or logn(x), where 'n' is the level of nesting. Let us solve the nested logarithm log2(log8(64)) First, we solve the inner logarithm, log8(64). Since 82 = 64, log8(64) = 2 Now, solving the outer logarithm, we get log2(2) = 1 Thus, log2(log8(64)) is simplified to 1. Simplify log5(log3(log7(343)))Solution:Here, log5(log3(log7(343)))Since 73 = 343, log7(343) = 3Solving the inner expression, we getlog5(log3(3))Since logb(b) = 1, log3(3) = 1Now, solving further logarithms, we getlog5(1) = 0 Last modified on May 28th, 2024 Nothing messes up an equation quite like logarithms. They are cumbersome, difficult to manipulate and a little mysterious for some people. Luckily, there's an easy way to rid your equation of these pesky mathematical expressions. All you have to do is remember that a logarithm is the inverse of an exponent. Although the base of a logarithm can be any number, the most common bases used in science are 10 and e, which is an irrational number known as Euler's number. To distinguish them, mathematicians use "log" when the base is 10 and "ln" when the base is e. To rid an equation of logarithms, raise both sides to the same exponent as the base of the logarithms. In equations with mixed terms, collect all the logarithms on one side and simplify first. The concept of a logarithm is simple, but it's a little difficult to put into words. A logarithm is the number of times you have to multiply a number by itself to get another number. Another way to say it is that a logarithm is the power to which a certain number – called the base – must be raised to get another number – called the argument of the logarithm. For example, log82 = 64 simply means that raising 8 to the power of 2 gives 64. In the equation log x = 100, the base is understood to be 10, and you can easily solve for the argument, x because it answers the question, "10 raised to what power equals 100?" The answer is 2. A logarithm is the inverse of an exponent, so the equation log x = 100 is another way of writing 10_x = 100. This relationship makes it possible to remove logarithms from an equation by raising both sides to the same exponent as the base of the logarithm. If the equation contains more than one logarithm, they must have the same base for this to work. In the simplest case, the logarithm of an unknown number equals another number: \(\log x = y\) Raise both sides to exponents of 10, and you get: \(10^{(\log x)} = 10^y\) Since 10(log x) is simply x, the equation becomes \(x = 10^y\) When all the terms in the equation are logarithms, raising both sides to an exponent produces a standard algebraic expression. For example, raise \(\log (x^2 - 1) = \log (x + 1)\) to a power of 10 and you get: \(x^2 – 1 = x + 1\) which simplifies to \(x^2 – x - 2 = 0.\) The solutions are x = −2; x = 1. In equations that contain a mixture of logarithms and other algebraic terms, it's important to collect all the logarithms on one side of the equation. You can then add or subtract terms. According to the law of logarithms, the following is true: \(\log x + \log y = \log (xy) \\ \log x - \log y = \log \bigg(\frac{x}{y}\bigg)\) Here's a procedure for solving an equation with mixed terms: Start with the equation: For example, \(\log x = \log (x - 2) + 3\) Rearrange the terms: \(\log x - \log (x - 2) = 3\) Apply the law of logarithms: \(\log \bigg(\frac{x}{x-2}\bigg) = 3\) Raise both sides to a power of 10: \(\frac{x}{x-2} = 10^3\) Solve for x: \(\bigg(\frac{x}{x-2}\bigg) = 10^3 = 1000x – 2000 \ -999x = -2000 \ x = \frac{2000}{999}=2.002\) Deziel, Chris. "How To Get Rid Of Logarithms" sciencing.com, . 27 October 2020. APA Deziel, Chris. (2020, October 27). How To Get Rid Of Logarithms. sciencing.com. Retrieved from Chicago Deziel, Chris. How To Get Rid Of Logarithms last modified August 30, 2022. In its simplest form, a logarithm answers the question: How many of one number multiply together to make another number? Example: How many 2s multiply together to make 8? Answer: 2 × 2 × 2 = 8, so we had to multiply 3 of the 2s to get 8 So the logarithm is 3 How to Write it We write it like this: log2(8) = 3 So these two things are the same: The number we multiply is called the "base", so we can say: "the logarithm of 8 with base 2 is 3" or "log base 2 of 8 is 3" or "the base-2 log of 8 is 3" Notice we are dealing with three numbers: the base: the number we are multiplying (a "2" in the example above) how often to use it in a multiplication (3 times, which is the logarithm) The number we want to get (an "8") More Examples We are asking "how many 5s need to be multiplied together to get 625?" 5 × 5 × 5 × 5 = 625, so we need 4 of the 5s Answer: log5(625) = 4 We are asking "how many 2s need to be multiplied together to get 64?" 2 × 2 × 2 × 2 × 2 × 2 = 64, so we need 6 of the 2s Answer: log2(64) = 6 Exponents Exponents and Logarithms are related, let's find out how ... The exponent says how many times to use the number in a multiplication. In this example: 23 = 2 × 2 × 2 = 8 (2 is used 3 times in a multiplication to get 8) So a logarithm answers a question like this: In this way: The logarithm tells us what the exponent is! In that example the "base" is 2 and the "exponent" is 3: So the logarithm answers the question: What exponent do we need (for one number to become another number)? The general case is: Example: What is log10(100) ... ? 102 = 100 So an exponent of 2 is needed to make 10 into 100, and: log10(100) = 2 Example: What is log3(81) ... ? 34 = 81 So an exponent of 4 is needed to make 3 into 81, and: log3(81) = 4 Common Logarithms: Base 10 Sometimes a logarithm is written without a base, like this: log(100) This usually means that the base is really 10. It is called a "common logarithm". Engineers love to use it. On a calculator it is the "log" button. It is how many times we need to use 10 in a multiplication, to get our desired number. Example: log(1000) = log10(1000) = 3 Natural Logarithms: Base "e" Another base that is often used is e (Euler's Number) which is about 2.71828. This is called a "natural logarithm". Mathematicians use this one a lot. On a calculator it is the "ln" button. It is how many times we need to use "e" in a multiplication, to get our desired number. Example: ln(7.389) = loge(7.389) ≈ 2 Because 2.718282 ≈ 7.389 But Sometimes There Is Confusion ... ! Mathematicians may use "log" (instead of "ln") to mean the natural logarithm. This can lead to confusion: Example Engineer Thinks Mathematician Thinks log(50) log10(50) loge(50) confusion ln(50) loge(50) loge(50) no confusion log10(50) log10(50) log10(50) no confusion So, be careful when you read "log" that you know what base they mean! Logarithms Can Have Decimals All of our examples have used whole number logarithms (like 2 or 3), but logarithms can have decimal values like 2.5, or 6.081, etc. Example: what is log10(26) ... ? Get your calculator, type in 26 and press log Answer is: 1.41497... The logarithm is saying that 101.41497... = 26 (10 with an exponent of 1.41497... equals 26) This is what it looks like on a graph: See how nice and smooth the line is. Read Logarithms Can Have Decimals to find out more. Negative Logarithms − Negative? But logarithms deal with multiplying. What is the opposite of multiplying? Dividing! A negative logarithm means how many times to divide by the number. We can have just one divide: Example: What is log8(0.125) ... ? Well, 1 ÷ 8 = 0.125, So log8(0.125) = −1 Or many divides: Example: What is log5(0.008) ... ? 1 ÷ 5 ÷ 5 ÷ 5 = 5−3, So log5(0.008) = −3 It All Makes Sense Multiplying and Dividing are all part of the same simple pattern. Let us look at some Base-10 logarithms as an example: Number How Many 10s Base-10 Logarithm .. etc.. 1000 1 × 10 × 10 × 10 log10(1000) = 3 100 1 × 10 × 10 log10(100) = 2 10 1 × 10 log10(10) = 1 1 1 log10(1) = 0 0.1 1 ÷ 10 log10(0.1) = −1 0.01 1 ÷ 10 ÷ 10 log10(0.01) = −2 0.001 1 ÷ 10 ÷ 10 ÷ 10 log10(0.001) = −3 .. etc.. Looking at that table, see how positive, zero or negative logarithms are really part of the same (fairly simple) pattern. The Word "Logarithm" is a word made up by Scottish mathematician John Napier (1550-1617), from the Middle Latin "logarithmus" meaning "ratio-number" 340, 341, 2384, 2385, 2386, 2387, 3181, 3182, 2388, 2389 Copyright © 2023 Rod Pierce Our online calculators, converters, randomizers, and content are provided "as is", free of charge, and without any warranty or guarantee. Each tool is carefully developed and rigorously tested, and our content is well-sourced, but despite our best efforts it is possible they contain errors. We are not to be held responsible for any resulting damages from proper or improper use of the service. See our full terms of service.Copyright © 2017-2025 GIGAcalculator.com Even before the development of calculus, mathematicians employed logarithms to convert division and multiplication problems into addition and subtraction issues. The power is raised to a specified number, usually a base number, in a logarithm to arrive at a specific number. Logarithms are effective in manipulating numbers of a size that is much easier to handle when you need to work with really huge numbers. The definition, formula, and functions will all be covered in-depth in this part along with several examples. In other words, the real integer y that has the property that x=by is the logarithm of x to base b. The symbol for the function is "logbx." (pronounced as "the logarithm of x to base b", "the base-b logarithm of x", or most commonly "the log, base b, of x"). Read More: Types of Probability Key Terms: Logarithmic function, Log, Exponential function, Domain, Range, [Click Here for Sample Questions] A logarithmic function is an inverse of the exponential function. In simple words, the logarithmic function can be defined as the inverse function to exponentiation. The logarithmic function is defined as For x>0, a>0 and a ≠ 1, y= logax if and only if x = ay Then the function is given by f(x) = loga x The logarithm base is equal to a. This can be understood as the log base of x. Base 10 and base e are the two most common bases used in logarithmic functions. The formula for a logarithmic function is f (x) = logb x. In this instance, the base of the logarithm is b, and the common bases used for natural logs and logs to base 10 are base and base 10. Numerous real-world uses of logarithmic functions can be found in fields including electronics, earthquake analysis, acoustics, and population forecasting. Read More: Value of Log 0 Graph of Logarithmic Function [Click Here for Sample Questions] Graph of Logarithmic Function Read More: Value of Log 1 Domain and Range of Logarithmic Function [Click Here for Sample Questions] A logarithmic function has real values larger than zero as its domain and real numbers as its range. In relation to the line y = x, the graph of y = logax and the graph of y = ax are symmetrical. Any function and its inverse are related in this way. The domain of y=ex is R+, i.e., x∈(0,∞) The range of y=ex is R i.e., x∈(-∞,∞) Read More: Value of Log 10 To Common and Natural Log [Click Here for Previous Year Questions] Common logarithmic function Common logarithmic function The logarithmic function It is denoted by log10 or simply log. f(x) = log10 x Natural Logarithmic Function - Different is a natural logarithm. A natural logarithm has the number e as its base when the base of the common logarithm is 10. Despite being a variable, e is actually a fixed, irrational number with a value of 2.718281828459. Other names for e include Euler's number and Napier's constant. To pay homage to mathematician Leonhard Euler, the letter e was chosen. Despite appearing difficult, e is a fascinating number. There are numerous uses for the function f(x) = loge x in business, economics, and biology. E has significance. The logarithmic function to the base e is called the natural logarithmic function and it is denoted by loge. f(x) = loge x Read More: Logarithmic Differentiation Properties of Logarithmic Function [Click Here for Sample Questions] All the properties of logarithmic functions are: Log(MN) = log(M) + log(N) This property denotes that the logarithm of a product is the sum of the logs of the multiplicands. Example: log 20 + log 2 = log 40 Log(M/N) = log(M) – log(N) This property denotes that the logarithm of a quotient is the difference between the log of the dividend and the divisor. Divide two numbers having the same base and subtract the exponent.Example: log 64 – log 6 = log (64/6) = log 64/6 Logb(M/N) = logb (M) – logb (N) This property denotes that the logarithm of a quotient is the difference between the log of the dividend and the divisor. Divide two numbers having the same base and subtract the exponent.Example: log 64 – log 9 = log (64/9) = 0 Loga 1 = 0 Logb (x) = ln x / ln b or logb (x) = log10 x / log10 b some other properties of logarithms are: logb (xy) = logb x + logb y Logb b = 1 Example: log1010 = 1 logb (x/y) = logb x - logb y logb (xy) = y logb x Logb xy = y logb x Logb x = logb y therefore x = y logb bx = x, Example: log1010x = x There are also several logarithmic identities. Properties of Logarithms Formula: This is the important virtue of allowing one to utilize the identities to get the log of a fraction. ln(ab)= ln(a)+ln(b) ln(ax) = x ln (a) Read More: Logarithm equations Solved Examples [Click Here for Sample Questions] Ques. 1. Express 2logx + 3logy – log z in logarithm form. Solution. 2logx + 3logy - log a = log a. Log x + logx2 + logy3 (By logarithmic rule- logab=b log a). LogX2y3 = log a (By the logarithmic rule, log (ab) = log a + log b) X2y3 = a. [If a = b, then loga = logb] Ques. 2. If log(x-1) + log(x+1) = log21, then find x. Ans. The answer is log(x-1)+log(x+1)=log21. log(x-1) + log(x+1) = 0 Because 1 Equals 0. (x-1) (x+1) = 1 X2 – 1=1 X2 = 2 x = ±√2 Because the log of a negative number is undefined, hence x= √2 Read More: Value of log infinity Things to Remember When a > 1, the logarithmic graph rises, and when 0 a 1, it falls. The domain is obtained by increasing the function's parameter above 0. a > 0 and a ≠ 1 The set of all real numbers is known as the range. The function is continuous and one-to-one. The graph and x-axis intersect at (1,0). The x-intercept is therefore 1. The equation y=logb(x+h)+k shifts the logarithmic function, y=logbx, by k units vertically and h units horizontally. The natural logarithm is a base-e logarithm. The symbol for it is lnx. The opposite of the natural logarithmic function, y=ln x. Previous Year Questions Ques. Express log(75/16)- 2log(5/9)+ log(32/243) in the terms of log 2 and log 3. (3 Marks) Ans. Express log(75/16) – 2log(5/9) + log(32/243). Given that log am - log n = log (am/n). log(75/16) + log(32/243) = log(75/16) – log(25/81) + log(32/243) log (75/16) ÷ (81/25)) + log(32/243) = log(243/16) + log(32/243) = log(243/16) × (32/243) = log 2 Ques. Express logam in terms of log a and log m. Ans. Since nlog am = log am = log (a/m)n = log(am)n. Logam = logam Ques. If logam=n, express am-1 in terms of a and n. (2 Marks) Ans. If logam=n, express am-1 in terms of a and n. Ans. am=n, am = m/a am-1 = m/a = m/a – n/1 = m/a Ques. If log5 (x-7) = 1, find x. (2 Marks) Ans. Provided, log5(x-7)=1 Logarithm rules allow us to write; 51 = x-7 5 = x-7 x=5+7 x=12 Ques. Find the log of 32 to the base 4. (2 Marks) Ans. Provided, log5(x-7)=1Logarithm rules allow us to write; 51 = x-7 5 = x-7 x=5+7 x=12 Ques. Find the log of 32 to the base 4. (2 Marks) Ans. log432 = x 4x = 32 (22)x = 2x2x2x2 22x = 25 2x =5 x=5/2 Ques. Log432 =5/2 Ques. Log101 = 0 should be expressed exponentially. (2 Marks) Ans. Assuming log101 = 0, we are aware of the law; logac=b = ab=c Hence, 100 = 1 Ques. Explain in a logarithmic form that 53 = 125. (2 Marks) Ans. 53 = 125 The fact is, ab= c loga c Hence, log5125 = 3 For Latest Updates on Upcoming Board Exams, Click Here: Check-Out: Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Mathematical function, inverse of an exponential function Plots of logarithm functions, with three commonly used bases. The special points logb b = 1 are indicated by dotted lines, and all curves intersect in logb 1 = 0. Arithmetic operationsvte Addition (+) term + term summand + summand addend + addend augend + addend } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{term}}\,+\,{\text{term}}\\\scriptstyle {\text{summand}}\,+\,{\text{summand}}\\\scriptstyle {\text{addend}}\,+\,{\text{addend}}\\\scriptstyle {\text{augend}}\,+\,{\text{addend}}\end{matrix}}\right\}\,=\,} sum {\displaystyle \scriptstyle {\text{sum}}} Subtraction (−) term − term minuend − subtrahend } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{term}}\,-\,{\text{term}}\\\scriptstyle {\text{minuend}}\,-\,{\text{subtrahend}}\end{matrix}}\right\}\,=\,} difference {\displaystyle \scriptstyle {\text{difference}}} Multiplication (×) factor × factor multiplier × multiplicand } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{factor}}\,\times \,{\text{factor}}\\\scriptstyle {\text{multiplier}}\,\times \,{\text{multiplicand}}\end{matrix}}\right\}\,=\,} product {\displaystyle \scriptstyle {\text{product}}} Division (÷) dividend divisor numerator denominator } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\frac {\scriptstyle {\text{dividend}}}{\scriptstyle {\text{divisor}}}}\\[1ex]\scriptstyle {\frac {\scriptstyle {\text{numerator}}}{\scriptstyle {\text{denominator}}}}\end{matrix}}\right\}\,=\,} { fraction quotient ratio {\displaystyle \scriptstyle \left\{{\begin{matrix}\scriptstyle {\text{fraction}}\\\scriptstyle {\text{quotient}}\\\scriptstyle {\text{ratio}}\end{matrix}}\right.} Exponentiation base exponent base power } = {\displaystyle \scriptstyle \left.{\begin{matrix}\scriptstyle {\text{base}}^{\text{exponent}}\\\scriptstyle {\text{base}}^{\text{power}}\end{matrix}}\right\}\,=\,} power {\displaystyle \scriptstyle {\text{power}}} nth root (√) radicand degree = {\displaystyle \scriptstyle {\sqrt[{\text{degree}}]{\scriptstyle {\text{radicand}}}}\,=\,} root {\displaystyle \scriptstyle {\text{root}}} Logarithm (log) log base (anti-logarithm) = {\displaystyle \scriptstyle \log _{\text{base}}({\text{anti-logarithm}})\,=\,} logarithm {\displaystyle \scriptstyle {\text{logarithm}}} vte In mathematics, the logarithm of a number is the exponent by which another fixed value, the base, must be raised to produce that number. For example, the logarithm of 1000 to base 10 is 3, because 1000 is 10 to the 3rd power: 1000 = 103 = 10 × 10 × 10. More generally, if x = by, then y is the logarithm of x to base b, written logb x, so log10 1000 = 3. As a single-variable function, the logarithm to base b is the inverse of exponentiation with base b. The logarithm base 10 is called the decimal or common logarithm and is commonly used in science and engineering. The natural logarithm has the number e ≈ 2.718 as its base; its use is widespread in mathematics and physics because of its very simple derivative. The binary logarithm uses base 2 and is widely used in computer science, information theory, music theory, and photography. When the base is unambiguous from the context or irrelevant it is often omitted, and the logarithm is written log x. Logarithms were introduced by John Napier in 1614 as a means of simplifying calculations.[1] They were rapidly adopted by navigators, scientists, engineers, surveyors, and others to perform high-accuracy computations more easily. Using logarithm tables, tedious multi-digit multiplication steps can be replaced by table look-ups and simpler addition. This is possible because the logarithm of a product is the sum of the logarithms of the factors: log b ⁡ (x y) = log b ⁡ x + log b ⁡ y , {\displaystyle \log _{b}(xy)=\log _{b}x+\log _{b}y,} provided that b, x and y are all positive and b ≠ 1. The slide rule, also based on logarithms, allows quick calculations without tables, but at lower precision. The present-day notion of logarithms comes from Leonhard Euler, who connected them to the exponential function in the 18th century, and who also introduced the letter e as the base of natural logarithms.[2] Logarithmic scales reduce wide-ranging quantities to smaller scopes. For example, the decibel (dB) is a unit used to express ratio as logarithms, mostly for signal power and amplitude (of which sound pressure is a common example). In chemistry, pH is a logarithmic measure for the acidity of an aqueous solution. Logarithms are commonplace in scientific formulae, and in measurements of the complexity of algorithms and of geometric objects called fractals. They help to describe frequency ratios of musical intervals, appear in formulas counting prime numbers or approximating factorials, inform some models in psychophysics, and can aid in forensic accounting. The concept of logarithm as the inverse of exponentiation extends to other mathematical structures as well. However, in general settings, the logarithm tends to be a multi-valued function. For example, the complex logarithm is the multi-valued inverse of the complex exponential function. Similarly, the discrete logarithm is the multi-valued inverse of the exponential function in finite groups; it has uses in public-key cryptography. The graph of the logarithm base 2 crosses the x-axis at x = 1 and passes through the points (2, 1), (4, 2), and (8, 3), depicting, e.g., log2(8) = 3 and 23 = 8. The graph gets arbitrarily close to the y-axis, but does not meet it. Addition, multiplication, and exponentiation are three of the most fundamental arithmetic operations. The inverse of addition is subtraction, and the inverse of multiplication is division. Similarly, a logarithm is the inverse operation of exponentiation. Exponentiation is when a number b, the base, is raised to a certain power y, the exponent, to give a value x; this is denoted by y = x. {\displaystyle b^{y}=x.} For example, raising 2 to the power of 3 gives 8: 2 3 = 8. {\displaystyle 2^{3}=8.} The logarithm of base b is the inverse operation, that provides the output y from the input x. That is, y = log b ⁡ x {\displaystyle y=\log _{b}x} is equivalent to x = b y {\displaystyle x=b^{y}} if b is a positive real number. (If b is not a positive real number, both exponentiation and logarithm can be defined but may take several values, which makes definitions much more complicated.) One of the main historical motivations of introducing logarithms is the formula log b ⁡ (x y) = log b ⁡ x + log b ⁡ y , {\displaystyle \log _{b}(xy)=\log _{b}x+\log _{b}y,} by which tables of logarithms allow multiplication and division to be reduced to addition and subtraction, a great aid to calculations before the invention of computers. Given a positive real number b such that b ≠ 1, the logarithm of a positive real number x with respect to base b[nb 1] is the exponent by which b must be raised to yield x. In other words, the logarithm of x to base b is the unique real number y such that b y = x {\displaystyle b^{y}=x}.[3] The logarithm is denoted "logb x" (pronounced as "the logarithm of x to base b", "the base-b logarithm of x", or most commonly "the log, base b, of x"). An equivalent and more succinct definition is that the function logb is the inverse function to the function x ↦ b x {\displaystyle x\mapsto b^{x}}. log2 16 = 4, since 24 = 2 × 2 × 2 × 2 = 16. Logarithms can also be negative: log 2 ⁡ 1 2 = − 1 {\textstyle \log _{2}\!{\frac {1}{2}}=-1} since 2 − 1 = 1 2 1 = 1 2 . {\textstyle 2^{-1}={\frac {1}{2^{1}}}={\frac {1}{2}}.} log10 150 is approximately 2.176, which lies between 2 and 3, just as 150 lies between 102 = 100 and 103 = 1000. For any base b, logb b = 1 and logb 1 = 0, since b1 = b and b0 = 1, respectively. Main article: List of logarithmic identities Several important formulas, sometimes called logarithmic identities or logarithmic laws, relate logarithms to one another.[4] The logarithm of a product is the sum of the logarithms of the numbers being multiplied; the logarithm of the ratio of two numbers is the difference of the logarithms. The logarithm of the p-th power of a number is p times the logarithm of the number itself; the logarithm of a p-th root is the logarithm of the number divided by p. The following table lists these identities with examples. Each of the identities can be derived after substitution of the logarithm definitions x = b log b ⁡ x {\displaystyle x=b^{\,\log _{b}x}} or y = b log b ⁡ y {\displaystyle y=b^{\,\log _{b}y}} in the left hand sides. In the following formulas, x {\displaystyle x} and y {\displaystyle y} are positive real numbers and p {\displaystyle p} is an integer greater than 1. Product, quotient, power, and root identities of logarithms Identity Formula Example Product log b ⁡ (x y) = log b ⁡ x + log b ⁡ y {\textstyle \log _{b}(xy)=\log _{b}x+\log _{b}y} log 3 ⁡ 243 = log 3 ⁡ (9 ⋅ 27) = log 3 ⁡ 9 + log 3 ⁡ 27 = 2 + 3 = 5 {\textstyle \log _{3}243=\log _{3}(9\cdot 27)=\log _{3}9+\log _{3}27=2+3=5} Quotient log b ⁡ x y = log b ⁡ x − log b ⁡ y {\textstyle \log _{b}\!{\frac {x}{y}}=\log _{b}x-\log _{b}y} log 2 ⁡ 16 = log 2 ⁡ 64 4 = log 2 ⁡ 64 − log 2 ⁡ 4 = 6 − 2 = 4 {\textstyle \log _{2}16=\log _{2}\!{\frac {64}{4}}=\log _{2}64-\log _{2}4=6-2=4} Power log b ⁡ (x p) = p log b ⁡ x {\textstyle \log _{b}\left(x^{p}\right)=p\log _{b}x} log 2 ⁡ 64 = log 2 ⁡ (2 6) = 6 log 2 ⁡ 2 = 6 {\textstyle \log _{2}64=\log _{2}\left(2^{6}\right)=6\log _{2}2=6} Root log b ⁡ x p = log b ⁡ x p {\textstyle \log _{b}{\sqrt[{p}]{x}}={\frac {\log _{b}x}{p}}} log 10 ⁡ 1000 = 1 2 log 10 ⁡ 1000 = 3 2 = 1.5 {\textstyle \log _{10}{\sqrt {1000}}={\frac {1}{2}}\log _{10}1000={\frac {3}{2}}=1.5} The logarithm logb x can be computed from the logarithms of x and b with respect to an arbitrary base k using the following formula:[nb 2] log b ⁡ x = log k ⁡ x log k ⁡ b . {\displaystyle \log _{b}x={\frac {\log _{k}x}{\log _{k}b}}.} Typical scientific calculators calculate the logarithms to bases 10 and e.[5] Logarithms with respect to any base b can be determined using either of these two logarithms by the previous formula: log b ⁡ x = log 10 ⁡ x log 10 ⁡ b = log e ⁡ x log e ⁡ b . {\displaystyle \log _{b}x={\frac {\log _{10}x}{\log _{10}b}}={\frac {\log _{e}x}{\log _{e}b}}.} Given a number x and its logarithm y = logb x to an unknown base b, the base is given by: b = x 1 y , {\displaystyle b=x^{\frac {1}{y}},} which can be seen from taking the defining equation x = b log b ⁡ x = b y {\displaystyle x=b^{\,\log _{b}x}=b^{y}} to the power of 1 y . {\displaystyle {\tfrac {1}{y}}.} Overlaid graphs of the logarithm for bases 1 2 , 2, and e Among all choices for the base, three are particularly common. These are b = 10, b = e (the irrational mathematical constant e ≈ 2.71828183), and b = 2 (the binary logarithm). In mathematical analysis, the logarithm base e is widespread because of analytical properties explained below. On the other hand, base 10 logarithms (the common logarithm) are easy to use for manual calculations in the decimal number system:[6] log 10 ⁡ (10 x) = log 10 ⁡ 10 + log 10 ⁡ x = 1 + log 10 ⁡ x . {\displaystyle \log _{10}\,(\,10\,x\,)\ =\;\log _{10}10\ +\;\log _{10}x\ =\ 1\,+\,\log _{10}x\,.} Thus, log10 (x) is related to the number of decimal digits of a positive integer x: The number of digits is the smallest integer strictly bigger than log10 (x) .[7] For example, log10(5986) is approximately 3.78 . The next integer above it is 4, which is the number of digits of 5986. Both the natural logarithm and the binary logarithm are used in information theory, corresponding to the use of nats or bits as the fundamental units of information, respectively.[8] Binary logarithms are also used in computer science, where the binary system is ubiquitous; in music theory, where a pitch ratio of two (the octave) is ubiquitous and the number of cents between any two pitches is a scaled version of the binary logarithm, or log 2 times 1200, of the pitch ratio (that is, 100 cents per semitone in conventional equal temperament), or equivalently the log base 21/1200 ; and in photography rescaled base 2 logarithms are used to measure exposure values, light levels, exposure times, lens apertures, and film speeds in "stops".[9] The abbreviation log x is often used when the intended base can be inferred based on the context or discipline, or when the base is indeterminate or immaterial. Common logarithms (base 10), historically used in logarithm tables and slide rules, are a basic tool for measurement and computation in many areas of science and engineering; in these contexts log x still often means the base ten logarithm.[10] In mathematics log x usually refers to the natural logarithm (base e).[11] In computer science and information theory, log often refers to binary logarithms (base 2).[12] The following table lists common notations for logarithms to these bases. The "ISO notation" column lists designations suggested by the International Organization for Standardization.[13] Base b Name for logb x ISO notation Other notations 2 binary logarithm lb x [14] ld x, log x, lg x,[15] log2 x e natural logarithm ln x [nb 3] log x, loge x 10 common logarithm lg x log x, log10 x b logarithm to base b logb x Main article: History of logarithms The history of logarithms in seventeenth-century Europe saw the discovery of a new function that extended the realm of analysis beyond the scope of algebraic methods. The method of logarithms was publicly propounded by John Napier in 1614, in a book titled Mirifici Logarithmorum Canonis Descriptio (Description of the Wonderful Canon of Logarithms).[19][20] Prior to Napier's invention, there had been other techniques of similar scopes, such as the prosthaphaeresis or the use of tables of progressions, extensively developed by Jost Bürgi around 1600.[21][22] Napier coined the term for logarithm in Middle Latin, logarithmus, literally meaning 'ratio-number', derived from the Greek logos 'proportion, ratio, word' + arithmos 'number'. The common logarithm of a number is the index of that power of ten which equals the number.[23] Speaking of a number as requiring so many figures is a rough allusion to common logarithm, and was referred to by Archimedes as the "order of a number".[24] The first real logarithms were heuristic methods to turn multiplication into addition, thus facilitating rapid computation. Some of these methods used tables derived from trigonometric identities.[25] Such methods are called prosthaphaeresis. Invention of the function now known as the natural logarithm began as an attempt to perform a quadrature of a rectangular hyperbola by Grégoire de Saint-Vincent, a Belgian Jesuit residing in Prague. Archimedes had written The Quadrature of the Parabola in the third century BC, but a quadrature for the hyperbola eluded all efforts until Saint-Vincent published his results in 1647. The relation that the logarithm provides between a geometric progression in its argument and an arithmetic progression of values, prompted A. A. de Sarasa to make the connection of Saint-Vincent's quadrature and the tradition of logarithms in prosthaphaeresis, leading to the term "hyperbolic logarithm", a synonym for natural logarithm. Soon the new function was appreciated by Christiaan Huygens, and James Gregory. The notation Log y was adopted by Gottfried Wilhelm Leibniz in 1675,[26] and the next year he connected it to the integral ∫ d y y . {\textstyle \int {\frac {dy}{y}}.} Before Euler developed his modern conception of complex natural logarithms, Roger Cotes had a nearly equivalent result when he showed in 1714 that[27] log ⁡ (cos ⁡ θ + i sin ⁡ θ) = i θ . {\displaystyle \log(\cos \theta +i\sin \theta)=i\theta .} The 1797 Encyclopædia Britannica explanation of logarithms By simplifying difficult calculations before calculators and computers became available, logarithms contributed to the advance of science, especially astronomy. They were critical to advances in surveying, celestial navigation, and other domains. Pierre-Simon Laplace called logarithms "...[a]n admirable artifice which, by reducing to a few days the labour of many months, doubles the life of the astronomer, and spares him the errors and disgust inseparable from long calculations."[28] As the function f(x) = bx is the inverse function of logb x, it has been called an antilogarithm.[29] Nowadays, this function is more commonly called an exponential function. A key tool that enabled the practical use of logarithms was the table of logarithms.[30] The first such table was compiled by Henry Briggs in 1617, immediately after Napier's invention but with the innovation of using 10 as the base. Briggs' first table contained the common logarithms of all integers in the range from 1 to 1000, with a precision of 14 digits. Subsequently, tables with increasing scope were written. These tables listed the values of log10 x for any number x in a certain range, at a certain precision. Base-10 logarithms were universally used for computation, hence the name common logarithm, since numbers that differ by factors of 10 have logarithms that differ by integers. The common logarithm of x can be separated into an integer part and a fractional part, known as the characteristic and mantissa. Tables of logarithms need only include the mantissa, as the characteristic can be easily determined by counting digits from the decimal point.[31] The characteristic of 10 · x is one plus the characteristic of x, and their mantissas are the same. Thus using a three-digit log table, the logarithm of 3542 is approximated by log 10 ⁡ 3542 = log 10 ⁡ (1000 ⋅ 3.542) = 3 + log 10 ⁡ 3.542 ≈ 3 + log 10 ⁡ 3.54 {\displaystyle {\begin{aligned}\log _{10}3542&=\log _{10}(1000\cdot 3.542)\\&=3+\log _{10}3.542\\&\approx 3+\log _{10}3.54\end{aligned}}} Greater accuracy can be obtained by interpolation: log 10 ⁡ 3542 ≈ 3 + log 10 ⁡ 3.54 + 0.2 (log 10 ⁡ 3.55 − log 10 ⁡ 3.54) {\displaystyle \log _{10}3542\approx {}3+\log _{10}3.54+0.2(\log _{10}3.55-\log _{10}3.54)} The value of 10x can be determined by reverse look up in the same table, since the logarithm is a monotonic function. The product and quotient of two positive numbers c and d were routinely calculated as the sum and difference of their logarithms. The product cd or quotient c/d came from looking up the antilogarithm of the sum or difference, via the same table: c d = 10 log 10 ⁡ c 10 log 10 ⁡ d = 10 log 10 ⁡ c + log 10 ⁡ d {\displaystyle cd=10^{\,\log _{10}c}\,10^{\,\log _{10}d}=10^{\,\log _{10}c\,+\,\log _{10}d}} and c d = c d − 1 = 10 log 10 ⁡ c − log 10 ⁡ d . {\displaystyle {\frac {c}{d}}=cd^{-1}=10^{\,\log _{10}c\,-\,\log _{10}d}.} For manual calculations that demand any appreciable precision, performing the lookups of the two logarithms, calculating their sum or difference, and looking up the antilogarithm is much faster than performing the multiplication by earlier methods such as prosthaphaeresis, which relies on trigonometric identities. Calculations of powers and roots are reduced to multiplications or divisions and lookups by c d = (10 log 10 ⁡ c) d = 10 d log 10 ⁡ c {\displaystyle c^{d}=\left(10^{\,\log _{10}c}\right)^{d}=10^{\,d\log _{10}c}} and c d = c 1 d = 10 1 d log 10 ⁡ c . {\displaystyle {\sqrt[{d}]{c}}=c^{\frac {1}{d}}=10^{{\frac {1}{d}}\log _{10}c}.} Trigonometric calculations were facilitated by tables that contained the common logarithms of trigonometric functions. Main article: Slide rule Another critical application was the slide rule, a pair of logarithmically divided scales used for calculation. The non-sliding logarithmic scale, Gunter's rule, was invented shortly after Napier's invention. William Oughtred enhanced it to create the slide rule—a pair of logarithmic scales movable with respect to each other. Numbers are placed on sliding scales at distances proportional to the differences between their logarithms. Sliding the upper scale appropriately amounts to mechanically adding logarithms, as illustrated here: Schematic depiction of a slide rule. Starting from 2 on the lower scale, add the distance to 3 on the upper scale to reach the product 6. The slide rule works because it is marked such that the distance from 1 to x is proportional to the logarithm of x. For example, adding the distance from 1 to 2 on the lower scale to the distance from 1 to 3 on the upper scale yields a product of 6, which is read off at the lower part. The slide rule was an essential calculating tool for engineers and scientists until the 1970s, because it allows, at the expense of precision, much faster computation than techniques based on tables.[32] A deeper study of logarithms requires the concept of a function. A function is a rule that, given one number, produces another number.[33] An example is the function producing the x-th power of b from any real number x, where the base b is a fixed number. This function is written as f(x) = b x. When b is positive and unequal to 1, we show below that f is invertible when considered as a function from the reals to the positive reals. Let b be a positive real number not equal to 1 and let f(x) = b x. It is a standard result in real analysis that any continuous strictly monotonic function is bijective between its domain and range. This fact follows from the intermediate value theorem.[34] Now, f is strictly increasing (for b > 1), or strictly decreasing (for 0 < b < 1),[35] is continuous, has domain R {\displaystyle \mathbb {R} } , and has range R > 0 {\displaystyle \mathbb {R} _{>0}} . Therefore, f is a bijection from R {\displaystyle \mathbb {R} } to R > 0 {\displaystyle \mathbb {R} _{>0}} . In other words, for each positive real number y, there is exactly one real number x such that b x = y {\displaystyle b^{x}=y} . We let log b : R > 0 → R {\displaystyle \log _{b}\colon \mathbb {R} _{>0}\to \mathbb {R} } denote the inverse of f. That is, logb y is the unique real number x such that b x = y {\displaystyle b^{x}=y} . This function is called the base-b logarithm function or logarithmic function (or just logarithm). The function logb x can also be essentially characterized by the product formula log b ⁡ (x y) = log b ⁡ x + log b ⁡ y . {\displaystyle \log _{b}(xy)=\log _{b}x+\log _{b}y.} More precisely, the logarithm to any base b > 1 is the only increasing function f from the positive reals to the reals satisfying f(b) = 1 and[36] f (x y) = f (x) + f (y) . {\displaystyle f(xy)=f(x)+f(y).} The graph of the logarithm function logb (x) (blue) is obtained by reflecting the graph of the function bx (red) at the diagonal line (x = y). As discussed above, the function logb is the inverse to the exponential function x ↦ b x {\displaystyle x\mapsto b^{x}} . Therefore, their graphs correspond to each other upon exchanging the x- and the y-coordinates (or upon reflection at the diagonal line x = y), as shown at the right: a point (t, u = bt) on the graph of f yields a point (u, t = logb u) on the graph of the logarithm and vice versa. As a consequence, logb (x) diverges to infinity (gets bigger than any given number) if x grows to infinity, provided that b is greater than one. In that case, logb(x) is an increasing function. For b < 1, logb (x) tends to minus infinity instead. When x approaches zero, logb x goes to minus infinity for b > 1 (plus infinity for b < 1, respectively). The graph of the natural logarithm (green) and its tangent at x = 1.5 (black) Analytic properties of functions pass to their inverses.[34] Thus, as f(x) = bx is a continuous and differentiable function, so is logb y. Roughly, a continuous function is differentiable if its graph has no sharp "corners". Moreover, as the derivative of f(x) evaluates to ln(b) bx by the properties of the exponential function, the chain rule implies that the derivative of logb x is given by[35][37] d d x log b ⁡ x = 1 x ln ⁡ b . {\displaystyle {\frac {d}{dx}}\log _{b}x={\frac {1}{x\ln b}}.} That is, the slope of the tangent touching the graph of the base-b logarithm at the point (x, logb (x)) equals 1/(x ln(b)). The derivative of ln(x) is 1/x; this implies that ln(x) is the unique antiderivative of 1/x that has the value 0 for x = 1. It is this very simple formula that motivated to qualify as "natural" the natural logarithm; this is also one of the main reasons of the importance of the constant e. The derivative with a generalized functional argument f(x) is d d x ln ⁡ f (x) = f ′ (x) f (x) . {\displaystyle {\frac {d}{dx}}\ln f(x)={\frac {f'(x)}{f(x)}}.} The quotient at the right hand side is called the logarithmic derivative of f. Computing f'(x) by means of the derivative of ln(f(x)) is known as logarithmic differentiation.[38] The antiderivative of the natural logarithm ln(x) is:[39] ∫ ln ⁡ (x) d x = x ln ⁡ (x) − x + C . {\displaystyle \int \ln(x)\,dx=x\ln(x)-x+C.} Related formulas, such as antiderivatives of logarithms to other bases can be derived from this equation using the change of bases.[40] The natural logarithm of t is the shaded area underneath the graph of the function f(x) = 1/x. The natural logarithm of t can be defined as the definite integral: ln ⁡ t = ∫ 1 t 1 x d x . {\displaystyle \ln t=\int _{1}^{t}{\frac {1}{x}}\,dx.} This definition has the advantage that it does not rely on the exponential function or any trigonometric functions; the definition is in terms of an integral of a simple reciprocal. As an integral, ln(t) equals the area between the x-axis and the graph of the function 1/x, ranging from x = 1 to x = t. This is a consequence of the fundamental theorem of calculus and the fact that the derivative of ln(x) is 1/x. Product and power logarithm formulas can be derived from this definition.[41] For example, the product formula ln(tu) = ln(t) + ln(u) is deduced as: ln ⁡ (t u) = ∫ 1 t u 1 x d x = (1) ∫ 1 t 1 x d x + ∫ t t u 1 x d x = (2) ln ⁡ (t) + ∫ 1 u 1 w d w = ln ⁡ (t) + ln ⁡ (u) . {\displaystyle {\begin{aligned}\ln(tu)&=\int _{1}^{tu}{\frac {1}{x}}\,dx\\&{\stackrel {(1)}{=}}\int _{1}^{t}{\frac {1}{x}}\,dx+\int _{t}^{tu}{\frac {1}{x}}\,dx\\&{\stackrel {(2)}{=}}\ln(t)+\int _{1}^{u}{\frac {1}{w}}\,dw\\&=\ln(t)+\ln(u).\end{aligned}}} The equality (1) splits the integral into two parts, while the equality (2) is a change of variable (w = x/t). In the illustration below, the splitting corresponds to dividing the area into the yellow and blue parts. Rescaling the left hand blue area vertically by the factor t and shrinking it by the same factor horizontally does not change its size. Moving it appropriately, the area fits the graph of the function f(x) = 1/x again. Therefore, the left hand blue area, which is the integral of f(x) from t to tu is the same as the integral from 1 to u. This justifies the equality (2) with a more geometric proof. A visual proof of the product formula of the natural logarithm The power formula ln(tr) = r ln(t) may be derived in a similar way: ln ⁡ (t r) = ∫ 1 t r 1 x d x = ∫ 1 t 1 w r (r w r − 1 d w) = r ∫ 1 t 1 w d w = r ln ⁡ (t) . {\displaystyle {\begin{aligned}\ln(t^{r})&=\int _{1}^{t^{r}}{\frac {1}{x}}dx\\&=\int _{1}^{t}{\frac {1}{w^{r}}}\left(rw^{r-1}\,dw\right)\\&=r\int _{1}^{t}{\frac {1}{w}}\,dw\\&=r\ln(t).\end{aligned}}} The second equality uses a change of variables (integration by substitution), w = x1/r. The sum over the reciprocals of natural numbers, 1 + 1 2 + 1 3 + ⋯ + 1 n = ∑ k = 1 n 1 k , {\displaystyle 1+{\frac {1}{2}}+{\frac {1}{3}}+\cdots +{\frac {1}{n}}=\sum _{k=1}^{n}{\frac {1}{k}},} is called the harmonic series. It is closely tied to the natural logarithm: as n tends to infinity, the difference, ∑ k = 1 n 1 k − ln ⁡ (n) , {\displaystyle \sum _{k=1}^{n}{\frac {1}{k}}-\ln(n),} converges (i.e. gets arbitrarily close) to a number known as the Euler–Mascheroni constant γ = 0.5772.... This relation aids in analyzing the performance of algorithms such as quicksort.[42] Real numbers that are not algebraic are called transcendental;[43] for example, π and e are such numbers, but 2 − 3 {\displaystyle {\sqrt {2-{\sqrt {3}}}}} is not. Almost all real numbers are transcendental. The logarithm is an example of a transcendental function. The Gelfond–Schneider theorem asserts that logarithms usually take transcendental, i.e. "difficult" values.[44] The logarithm keys (LOG for base 10 and LN for base e) on a TI-83 Plus graphing calculator Logarithms are easy to compute in some cases, such as log10(1000) = 3. In general, logarithms can be calculated using power series or the arithmetic–geometric mean, or be retrieved from a precalculated logarithm table that provides a fixed precision.[45][46] Newton's method, an iterative method to solve equations approximately, can also be used to calculate the logarithm, because its inverse function, the exponential function, can be computed efficiently.[47] Using look-up tables, CORDIC-like methods can be used to compute logarithms by using only the operations of addition and bit shifts.[48][49] Moreover, the binary logarithm algorithm calculates lb(x) recursively, based on repeated squarings of x, taking advantage of the relation log 2 ⁡ (x 2) = 2 log 2 ⁡ | x | . {\displaystyle \log _{2}\left(x^{2}\right)=2\log _{2}|x|.} The Taylor series of ln(z) centered at z = 1. The animation shows the first 10 approximations along with the 99th and 100th. The approximations do not converge beyond a distance of 1 from the center. For any real number z that satisfies 0 < z ≤ 2, the following formula holds:[nb 4][50] ln ⁡ (z) = (z − 1) 1 1 − (z − 1) 2 2 + (z − 1) 3 3 − (z − 1) 4 4 + ⋯ = ∑ k = 1 ∞ (− 1) k + 1 (z − 1) k k . {\displaystyle {\begin{aligned}\ln(z)&={\frac {(z-1)^{1}}{1}}-{\frac {(z-1)^{2}}{2}}+{\frac {(z-1)^{3}}{3}}-{\frac {(z-1)^{4}}{4}}+\cdots \\&=\sum _{k=1}^{\infty }(-1)^{k+1}{\frac {(z-1)^{k}}{k}}.\end{aligned}}} Equating the function ln(z) to this infinite sum (series) is shorthand for saying that the function can be approximated to a more and more accurate value by the following expressions (known as partial sums): (z − 1) , (z − 1) − (z − 1) 2 2 , (z − 1) − (z − 1) 2 2 + (z − 1) 3 3 , … {\displaystyle (z-1),\ \ (z-1)-{\frac {(z-1)^{2}}{2}},\ \ (z-1)-{\frac {(z-1)^{2}}{2}}+{\frac {(z-1)^{3}}{3}},\ \ldots } For example, with z = 1.5 the third approximation yields 0.4167, which is about 0.011 greater than ln(1.5) = 0.405465, and the ninth approximation yields 0.40553, which is only about 0.0001 greater. The nth partial sum can approximate ln(z) with arbitrary precision, provided the number of summands n is large enough. In elementary calculus, the series is said to converge to the function ln(z), and the function is the limit of the series. It is the Taylor series of the natural logarithm at z = 1. The Taylor series of ln(z) provides a particularly useful approximation to ln(1 + z) when z is small, |z| < 1, since then ln ⁡ (1 + z) = z − z 2 2 + z 3 3 − ⋯ ≈ z . {\displaystyle \ln(1+z)=z-{\frac {z^{2}}{2}}+{\frac {z^{3}}{3}}-\cdots \approx z.} For example, with z = 0.1 the first-order approximation gives ln(1.1) ≈ 0.1, which is less than 5% off the correct value 0.0953. Another series is based on the inverse hyperbolic tangent function: ln ⁡ (z) = 2 ⋅ artanh ⁡ z − 1 z + 1 = 2 (z − 1 z + 1 + 1 3 (z − 1 z + 1) 3 + 1 5 (z − 1 z + 1) 5 + ⋯) , {\displaystyle \ln(z)=2\cdot \operatorname {artanh} \,{\frac {z-1}{z+1}}=2\left({\frac {z-1}{z+1}}+{\frac {1}{3}}{\left({\frac {z-1}{z+1}}\right)}^{3}+{\frac {1}{5}}{\left({\frac {z-1}{z+1}}\right)}^{5}+\cdots \right),} for any real number z > 0.[nb 5][50] Using sigma notation, this is also written as ln ⁡ (z) = 2 ∑ k = 0 ∞ 1 2 k + 1 (z − 1 z + 1) 2 k + 1 . {\displaystyle \ln(z)=2\sum _{k=0}^{\infty }{\frac {1}{2k+1}}\left({\frac {z-1}{z+1}}\right)^{2k+1}.} This series can be derived from the above Taylor series. It converges quicker than the Taylor series, especially if z is close to 1. For example, for z = 1.5, the first three terms of the second series approximate ln(1.5) with an error of about 3×10−6. The quick convergence for z close to 1 can be taken advantage of in the following way: given a low-accuracy approximation y ≈ ln(z) and putting A = z exp ⁡ (y) , {\displaystyle A={\frac {z}{\exp(y)}},} the logarithm of z is: ln ⁡ (z) = y + ln ⁡ (A) . {\displaystyle \ln(z)=y+\ln(A).} The better the initial approximation y is, the closer A is to 1, so its logarithm can be calculated efficiently. A can be calculated using the exponential series, which converges quickly provided y is not too large. Calculating the logarithm of larger z can be reduced to smaller values of z by writing z = a · 10b, so that ln(z) = ln(a) + b · ln(10). A closely related method can be used to compute the logarithm of integers. Putting z = n + 1 n {\displaystyle \textstyle z={\frac {n+1}{n}}} in the above series, it follows that: ln ⁡ (n + 1) = ln ⁡ (n) + 2 ∑ k = 0 ∞ 1 2 k + 1 (1 2 n + 1) 2 k + 1 . {\displaystyle \ln(n+1)=\ln(n)+2\sum _{k=0}^{\infty }{\frac {1}{2k+1}}\left({\frac {1}{2n+1}}\right)^{2k+1}.} If the logarithm of a large integer n is known, then this series yields a fast converging series for log(n+1), with a rate of convergence of (1 2 n + 1) 2 {\textstyle \left({\frac {1}{2n+1}}\right)^{2}} . The arithmetic–geometric mean yields high-precision approximations of the natural logarithm. Sasaki and Kanada showed in 1982 that it was particularly fast for precisions between 400 and 1000 decimal places, while Taylor series methods were typically faster when less precision was needed. In their work ln(x) is approximated to a precision of 2−p (or p precise bits) by the following formula (due to Carl Friedrich Gauss):[51][52] ln ⁡ (x) ≈ π 2 M ⁡ (1 , 2 2 − m / x) − m ln ⁡ (2) . {\displaystyle \ln(x)\approx {\frac {\pi }{2\,\mathrm {M} \!\left(1,2^{2-m}/x\right)}}-m\ln(2).} Here M(x, y) denotes the arithmetic–geometric mean of x and y. It is obtained by repeatedly calculating the average (x + y)/2 (arithmetic mean) and x y {\textstyle {\sqrt {xy}}} (geometric mean) of x and y then let those two numbers become the next x and y. The two numbers quickly converge to a common limit which is the value of M(x, y). m is chosen such that x 2 m > 2 p / 2 . {\displaystyle x\,2^{m}>2^{p/2}.\,} to ensure the required precision. A larger m makes the M(x, y) calculation take more steps (the initial x and y are farther apart so it takes more steps to converge) but gives more precision. The constants π and ln(2) can be calculated with quickly converging series. While at Los Alamos National Laboratory working on the Manhattan Project, Richard Feynman developed a bit-processing algorithm to compute the logarithm that is similar to long division and was later used in the Connection Machine. The algorithm relies on the fact that every real number x where 1 < x < 2 can be represented as a product of distinct factors of the form 1 + 2−k. The algorithm sequentially builds that product P, starting with P = 1 and k = 1: if P · (1 + 2−k) < x, then it changes P to P · (1 + 2−k). It then increases k {\displaystyle k} by one regardless. The algorithm stops when k is large enough to give the desired accuracy. Because log(x) is the sum of the terms of the form log(1 + 2−k) corresponding to those k for which the factor 1 + 2−k was included in the product P, log(x) may be computed by simple addition, using a table of log(1 + 2−k) for all k. Any base may be used for the logarithm table.[53] A nautilus shell displaying a logarithmic spiral Logarithms have many applications inside and outside mathematics. Some of these occurrences are related to the notion of scale invariance. For example, each chamber of the shell of a nautilus is an approximate copy of the next one, scaled by a constant factor. This gives rise to a logarithmic spiral.[54] Benford's law on the distribution of leading digits can also be explained by scale invariance.[55] Logarithms are also linked to self-similarity. For example, logarithms appear in the analysis of algorithms that solve a problem by dividing it into two similar smaller problems and patching their solutions.[56] The dimensions of self-similar geometric shapes, that is, shapes whose parts resemble the overall picture are also based on logarithms. Logarithmic scales are useful for quantifying the relative change of a value as opposed to its absolute difference. Moreover, because the logarithmic function log(x) grows very slowly for large x, logarithmic scales are used to compress large-scale scientific data. Logarithms also occur in numerous scientific formulas, such as the Tsiolkovsky rocket equation, the Fenske equation, or the Nernst equation. Main article: Logarithmic scale A logarithmic chart depicting the value of one Goldmark in Papiermarks during the German hyperinflation in the 1920s Scientific quantities are often expressed as logarithms of other quantities, using a logarithmic scale. For example, the decibel is a unit of measurement associated with logarithmic-scale quantities. It is based on the common logarithm of ratios—10 times the common logarithm of a power ratio or 20 times the common logarithm of a voltage ratio. It is used to quantify the attenuation or amplification of electrical signals,[57] to describe power levels of sounds in acoustics,[58] and the absorbance of light in the fields of spectrometry and optics. The signal-to-noise ratio describing the amount of unwanted noise in relation to a (meaningful) signal is also measured in decibels.[59] In a similar vein, the peak signal-to-noise ratio is commonly used to assess the quality of sound and image compression methods using the logarithm.[60] The strength of an earthquake is measured by taking the common logarithm of the energy emitted at the quake. This is used in the moment magnitude scale or the Richter magnitude scale. For example, a 5.0 earthquake releases 32 times (101.5) and a 6.0 releases 1000 times (103) the energy of a 4.0.[61] Apparent magnitude measures the brightness of stars logarithmically.[62] In chemistry the negative of the decimal logarithm, the decimal cologarithm, is indicated by the letter p.[63] For instance, pH is the decimal cologarithm of the activity of hydronium ions (the form hydrogen ions H+ take in water).[64] The activity of hydronium ions in neutral water is 10−7 mol·L−1, hence a pH of 7. Vinegar typically has a pH of about 3. The difference of 4 corresponds to a ratio of 104 of the activity, that is, vinegar's hydronium ion activity is about 10−3 mol·L−1. Semilog (log–linear) graphs use the logarithmic scale concept for visualization: one axis, typically the vertical one, is scaled logarithmically. For example, the chart at the right compresses the steep increase from 1 million to 1 trillion to the same space (on the vertical axis) as the increase from 1 to 1 million. In such graphs, exponential functions of the form f(x) = a · bx appear as straight lines with slope equal to the logarithm of b. Log-log graphs scale both axes logarithmically, which causes functions of the form f(x) = a · xk to be depicted as straight lines with slope equal to the exponent k. This is applied in visualizing and analyzing power laws.[65] Logarithms occur in several laws describing human perception:[66][67] Hick's law proposes a logarithmic relation between the time individuals take to choose an alternative and the number of choices they have.[68] Fitts's law predicts that the time required to rapidly move to a target area is a logarithmic function of the ratio between the distance to a target and the size of the target.[69] In psychophysics, the Weber–Fechner law proposes a logarithmic relationship between stimulus and sensation such as the actual vs. the perceived weight of an item a person is carrying.[70] (This "law", however, is less realistic than more recent models, such as Stevens's power law.[71]) Psychological studies found that individuals with little mathematics education tend to estimate quantities logarithmically, that is, they position a number on an unmarked line according to its logarithm, so that 10 is positioned as close to 100 as 100 is to 1000. Increasing education shifts this to a linear estimate (positioning 1000 10 times as far away) in some circumstances, while logarithms are used when the numbers to be plotted are difficult to plot linearly.[72][73] Three probability density functions (PDF) of random variables with log-normal distributions. The location parameter μ, which is zero for all three of the PDFs shown, is the mean of the logarithm of the random variable, not the mean of the variable itself. Distribution of first digits (in %, red bars) in the population of the 237 countries of the world. Black dots indicate the distribution predicted by Benford's law. Logarithms arise in probability theory: the law of large numbers dictates that, for a fair coin, as the number of coin-tosses increases to infinity, the observed proportion of heads approaches one-half. The fluctuations of this proportion about one-half are described by the law of the iterated logarithm.[74] Logarithms also occur in log-normal distributions. When the logarithm of a random variable has a normal distribution, the variable is said to have a log-normal distribution.[75] Log-normal distributions are encountered in many fields, wherever a variable is formed as the product of many independent positive random variables, for example in the study of turbulence.[76] Logarithms are used for maximum-likelihood estimation of parametric statistical models. For such a model, the likelihood function depends on at least one parameter that must be estimated. A maximum of the likelihood function occurs at the same parameter-value as a maximum of the logarithm of the likelihood (the "log likelihood"), because the logarithm is an increasing function. The log-likelihood is easier to maximize, especially for the multiplied likelihoods for independent random variables.[77] Benford's law describes the occurrence of digits in many data sets, such as heights of buildings. According to Benford's law, the probability that the first decimal-digit of an item in the data sample is d (from 1 to 9) equals log10 (d + 1) − log10 (d), regardless of the unit of measurement.[78] Thus, about 30% of the data can be expected to have 1 as first digit, 18% start with 2, etc. Auditors examine deviations from Benford's law to detect fraudulent accounting.[79] The logarithm transformation is a type of data transformation used to bring the empirical distribution closer to the assumed one. Analysis of algorithms is a branch of computer science that studies the performance of algorithms (computer programs solving a certain problem).[80] Logarithms are valuable for describing algorithms that divide a problem into smaller ones, and join the solutions of the subproblems.[81] For example, to find a number in a sorted list, the binary search algorithm checks the middle entry and proceeds with the half before or after the middle entry if the number is still not found. This algorithm requires, on average, log2 (N) comparisons, where N is the list's length.[82] Similarly, the merge sort algorithm sorts an unsorted list by dividing the list into halves and sorting these first before merging the results. Merge sort algorithms typically require a time approximately proportional to N · log(N).[83] The base of the logarithm is not specified here, because the result only changes by a constant factor when another base is used. A constant factor is usually disregarded in the analysis of algorithms under the standard uniform cost model.[84] A function f(x) is said to grow logarithmically if f(x) is (exactly or approximately) proportional to the logarithm of x. (Biological descriptions of organism growth, however, use this term for an exponential function.[85]) For example, any natural number N can be represented in binary form in no more than log2 N + 1 bits. In other words, the amount of memory needed to store N grows logarithmically with N. Billiards on an oval billiard table. Two particles, starting at the center with an angle differing by one degree, take paths that diverge chaotically because of reflections at the boundary. Entropy is broadly a measure of the disorder of some system. In statistical thermodynamics, the entropy S of some physical system is defined as S = − k ∑ i p i ln ⁡ (p i) . {\displaystyle S=-k\sum

_{i}p_{i}\ln(p_{i}).\,} The sum is over all possible states i of the system in question, such as the positions of gas particles in a container. Moreover, pi is the probability that the state i is attained and k is the Boltzmann constant. Similarly, entropy in information theory measures the quantity of information. If a message recipient may expect any one of N possible messages with equal likelihood, then the amount of information conveyed by any one such message is quantified as log2 N bits.[86] Lyapunov exponents use logarithms to gauge the degree of chaoticity of a dynamical system. For example, for a particle moving on an oval billiard table, even small changes of the initial conditions result in very different paths of the particle. Such systems are chaotic in a deterministic way, because small measurement errors of the initial state predictably lead to largely different final states.[87] At least one Lyapunov exponent of a deterministically chaotic system is positive. The Sierpinski triangle (at the right) is constructed by repeatedly replacing equilateral triangles by three smaller ones. Logarithms occur in definitions of the dimension of fractals.[88] Fractals are geometric objects that are self-similar in the sense that small parts reproduce, at least roughly, the entire global structure. The Sierpinski triangle (pictured) can be covered by three copies of itself, each having sides half the original length. This makes the Hausdorff dimension of this structure ln(3)/ln(2) ≈ 1.58. Another logarithm-based notion of dimension is obtained by counting the number of boxes needed to cover the fractal in question. Four different octaves shown on a linear scale, then shown on a logarithmic scale (as the ear hears them) Logarithms are related to musical tones and intervals. In equal temperament tunings, the frequency ratio depends only on the interval between two tones, not on the specific frequency, or pitch, of the individual tones. In the 12-tone equal temperament tuning common in modern Western music, each octave (doubling of frequency) is broken into twelve equally spaced intervals called semitones. For example, if the note A has a frequency of 440 Hz then the note B-flat has a frequency of 466 Hz. The interval between A and B-flat is a semitone, as is the one between B-flat and B (frequency 493 Hz). Accordingly, the frequency ratios agree: 466 440 ≈ 493 466 ≈ 1.059 ≈ 2 12 . {\displaystyle {\frac {466}{440}}\approx {\frac {493}{466}}\approx 1.059\approx {\sqrt[{12}]{2}}.} Intervals between arbitrary pitches can be measured in octaves by taking the base-2 logarithm of the frequency ratio, can be measured in equally tempered semitones by taking the base-21/12 logarithm (12 times the base-2 logarithm), or can be measured in cents, hundredths of a semitone, by taking the base-21/1200 logarithm (1200 times the base-2 logarithm). The latter is used for finer encoding, as it is needed for finer measurements or non-equal temperaments.[89] Interval (the two tones are played at the same time) 1/12 tone play Semitone play Just major third play Major third play Tritone play Octave play Frequency ratio r {\displaystyle r} 2 1 72 = 1.0097 {\displaystyle 2^{\frac {1}{72}}\approx 1.0097} 2 1 12 = 1.0595 {\displaystyle 2^{\frac {1}{12}}\approx 1.0595} 5 4 = 1.25 {\displaystyle {\tfrac {5}{4}}=1.25} 2 4 12 = 2 3 = 1.2599 {\displaystyle {\begin{aligned}2^{\frac {4}{12}}&={\sqrt[{3}]{2}}\\&\approx 1.2599\end{aligned}}} 2 6 12 = 2 = 1.4142 {\displaystyle {\begin{aligned}2^{\frac {6}{12}}&={\sqrt {2}}\\&\approx 1.4142\end{aligned}}} 2 12 12 = 2 {\displaystyle 2^{\frac {12}{12}}=2} Number of semitones 12 log 2 r {\displaystyle 12\log _{2}r} 1 6 {\displaystyle {\tfrac {1}{6}}} 1 {\displaystyle 1} ≈ 3.8631 {\displaystyle \approx 3.8631} 4 {\displaystyle 4} 6 {\displaystyle 6} 12 {\displaystyle 12} Number of cents 1200 log 2 r {\displaystyle 1200\log _{2}r} 16 2 3 {\displaystyle 16{\tfrac {2}{3}}} 100 {\displaystyle 100} ≈ 386.31 {\displaystyle \approx 386.31} 400 {\displaystyle 400} 600 {\displaystyle 600} 1200 {\displaystyle 1200} Natural logarithms are closely linked to counting prime numbers (2, 3, 5, 7, 11, ...), an important topic in number theory. For any integer x, the quantity of prime numbers less than or equal to x is denoted π(x). The prime number theorem asserts that π(x) is approximately given by x ln (x) , {\displaystyle {\frac {x}{\ln(x)}},} in the sense that the ratio of π(x) and that fraction approaches 1 when x tends to infinity.[90] As a consequence, the probability that a randomly chosen number between 1 and x is prime is inversely proportional to the number of decimal digits of x. A far better estimate of π(x) is given by the offset logarithmic integral function Li(x), defined by L i (x) = ∫ 2 x 1 ln (t) d t . {\displaystyle \mathrm {Li} (x)=\int _{2}^{x}{\frac {1}{\ln(t)}}\,dt.} The Riemann hypothesis, one of the oldest open mathematical conjectures, can be stated in terms of comparing π(x) and Li(x).[91] The Erdős–Kac theorem describing the number of distinct prime factors also involves the natural logarithm. The logarithm of n factorial, n! = 1 · 2 · ... · n, is given by ln (n !) = ln (1) + ln (2) + ⋯ + ln (n) . {\displaystyle \ln(n!)=\ln(1)+\ln(2)+\cdots +\ln(n).} This can be used to obtain Stirling's formula, an approximation of n! for large n.[92] Main article: Complex logarithm Polar form of z = x + iy. Both φ and φ' are arguments of z. All the complex numbers a that solve the equation e a = z {\displaystyle e^{a}=z} are called complex logarithms of z, when z is (considered as) a complex number. A complex number is commonly represented as z = x + iy, where x and y are real numbers and i is an imaginary unit, the square of which is −1. Such a number can be visualized by a point in the complex plane, as shown at the right. The polar form encodes a non-zero complex number z by its absolute value, that is, the (positive, real) distance r to the origin, and an angle between the real (x) axis Re and the line passing through both the origin and z. This angle is called the argument of z. The absolute value r of z is given by r = x 2 + y 2 . {\displaystyle \textstyle r={\sqrt {x^{2}+y^{2}}}.} Using the geometrical interpretation of sine and cosine and their periodicity in 2π, any complex number z may be denoted as z = x + i y = r (cos ⁡ φ + i sin ⁡ φ) = r (cos ⁡ (φ + 2 k π) + i sin ⁡ (φ + 2 k π)) , {\displaystyle {\begin{aligned}z&=x+iy\\&=r(\cos \varphi +i\sin \varphi)\\&=r(\cos(\varphi +2k\pi)+i\sin(\varphi +2k\pi)),\end{aligned}}} for any integer number k. Evidently the argument of z is not uniquely specified: both φ and φ' = φ + 2kπ are valid arguments of z for all integers k, because adding 2kπ radians or k⋅360°[nb 6] to φ corresponds to "winding" around the origin counter-clock-wise by k turns. The resulting complex number is always z, as illustrated at the right for k = 1. One may select exactly one of the possible arguments of z as the so-called principal argument, denoted Arg(z), with a capital A, by requiring φ to belong to one, conveniently selected turn, e.g. −π < φ ≤ π[93] or 0 ≤ φ < 2π.[94] These regions, where the argument of z is uniquely determined are called branches of the argument function. The principal branch (-π, π) of the complex logarithm, Log(z). The black point at z = 1 corresponds to absolute value zero and brighter colors refer to bigger absolute values. The hue of the color encodes the argument of Log(z). Euler's formula connects the trigonometric functions sine and cosine to the complex exponential: e i φ = cos ⁡ φ + i sin ⁡ φ . {\displaystyle e^{i\varphi }=\cos \varphi +i\sin \varphi .} Using this formula, and again the periodicity, the following identities hold:[95] z = r (cos ⁡ φ + i sin ⁡ φ) = r (cos ⁡ (φ + 2 k π) + i sin ⁡ (φ + 2 k π)) = r e i (φ + 2 k π) = e ln ⁡ (r) e i (φ + 2 k π) = e ln ⁡ (r) + i (φ + 2 k π) = e a k , {\displaystyle {\begin{aligned}z&=r\left(\cos \varphi +i\sin \varphi \right)\\&=r\left(\cos(\varphi +2k\pi)+i\sin(\varphi +2k\pi)\right)\\&=re^{i(\varphi +2k\pi)}\\&=e^{\ln(r)}e^{i(\varphi +2k\pi)}\\&=e^{\ln(r)+i(\varphi +2k\pi)}=e^{a_{k}},\end{aligned}}} where ln(r) is the unique real natural logarithm, ak denote the complex logarithms of z, and k is an arbitrary integer. Therefore, the complex logarithms of z, which are all those complex values ak for which the ak-th power of e equals z, are the infinitely many values a k = ln ⁡ (r) + i (φ + 2 k π) , {\displaystyle a_{k}=\ln(r)+i(\varphi +2k\pi),} for arbitrary integers k. Taking k such that φ + 2kπ is within the defined interval for the principal arguments, then ak is called the principal value of the logarithm, denoted Log(z), again with a capital L. The principal argument of any positive real number x is 0; hence Log(x) is a real number and equals the real (natural) logarithm. However, the above formulas for logarithms of products and powers do not generalize to the principal value of the complex logarithm.[96] The illustration at the right depicts Log(z), confining the arguments of z to the interval (−π, π]. This way the corresponding branch of the complex logarithm has discontinuities all along the negative real x axis, which can be seen in the jump in the hue there. This discontinuity arises from jumping to the other boundary in the same branch, when crossing a boundary, i.e. not changing to the corresponding k-value of the continuously neighboring branch. Such a locus is called a branch cut. Dropping the range restrictions on the argument makes the relations "argument of z", and consequently the "logarithm of z", multi-valued functions. Exponentiation occurs in many areas of mathematics and its inverse function is often referred to as the logarithm. For example, the logarithm of a matrix is the (multi-valued) inverse function of the matrix exponential.[97] Another example is the p-adic logarithm, the inverse function of the p-adic exponential. Both are defined via Taylor series analogous to the real case.[98] In the context of differential geometry, the exponential map maps the tangent space at a point of a manifold to a neighborhood of that point. Its inverse is also called the logarithmic (or log) map.[99] In the context of finite groups exponentiation is given by repeatedly multiplying one group element b with itself. The discrete logarithm is the integer n solving the equation b n = x , {\displaystyle b^{n}=x,} where x is an element of the group. Carrying out the exponentiation can be done efficiently, but the discrete logarithm is believed to be very hard to calculate in some groups. This asymmetry has important applications in public key cryptography, such as for example in the Diffie–Hellman key exchange, a routine that allows secure exchanges of cryptographic keys over unsecured information channels.[100] Zech's logarithm is related to the discrete logarithm in the multiplicative group of non-zero elements of a finite field.[101] Further logarithm-like inverse functions include the double logarithm ln(ln(x)), the super- or hyper-4-logarithm (a slight variation of which is called iterated logarithm in computer science), the Lambert W function, and the logit. They are the inverse functions of the double exponential function, tetration, of f(w) = wew,[102] and of the logistic function, respectively.[103] From the perspective of group theory, the identity log(cd) = log(c) + log(d) expresses a group isomorphism between positive reals under multiplication and reals under addition. Logarithmic functions are the only continuous isomorphisms between these groups.[104] By means of that isomorphism, the Haar measure (Lebesgue measure) dx on the reals corresponds to the Haar measure dx/x on the positive reals.[105] The non-negative reals not only have a multiplication, but also have addition, and form a semiring, called the probability semiring; this is in fact a semifield. The logarithm then takes multiplication to addition (log multiplication), and takes addition to log addition (LogSumExp), giving an isomorphism of semirings between the probability semiring and the log semiring. Logarithmic one-forms df/f appear in complex analysis and algebraic geometry as differential forms with logarithmic poles.[106] The polylogarithm is the function defined by Li s ⁡ (z) = ∑ k = 1 ∞ z k k s . {\displaystyle \operatorname {Li} _{s}(z)=\sum _{k=1}^{\infty }{z^{k} \over k^{s}}.} It is related to the natural logarithm by Li1 (z) = −ln(1 − z). Moreover, Lis (1) equals the Riemann zeta function ζ(s).[107] Mathematics portalArithmetic portalChemistry portalGeography portalEngineering portal Decimal exponent (dex) Exponential function Index of logarithm articles ^ The restrictions on x and b are explained in the section "Analytic properties". ^ Proof: Taking the logarithm to base k of the defining identity x = b log b ⁡ x , {\textstyle x=b^{\log _{b}x},} one gets log k ⁡ x = log k ⁡ (b log b ⁡ x) = log b ⁡ x ⋅ log k ⁡ b . {\displaystyle \log _{k}x=\log _{k}\left(b^{\log _{b}x}\right)=\log _{b}x\cdot \log _{k}b.} The formula follows by solving for log b ⁡ x . {\displaystyle \log _{b}x.} ^ z Some mathematicians disapprove of this notation. In his 1985 autobiography, Paul Halmos criticized what he considered the "childish ln notation", which he said no mathematician had ever used.[16] The notation was invented by the 19th century mathematician I. Stringham.[17][18] ^ The same series holds for the principal value of the complex logarithm for complex numbers z satisfying |z − 1| < 1. ^ The same series holds for the principal value of the complex logarithm for complex numbers z with positive real part. ^ See radian for the conversion between 2π and 360 degree.
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Media related to Logarithm at Wikimedia Commons The dictionary definition of logarithm at Wiktionary Quotations related to History of logarithms at Wikiquote A lesson on logarithms can be found on Wikiversity Weisstein, Eric W., "Logarithm", MathWorld Khan Academy: Logarithms, free online micro lectures "Logarithmic function", Encyclopedia of Mathematics, EMS Press, 2001 [1994] Colin Byfleet, Educational video on logarithms, retrieved 12 October 2010 Edward Wright, Translation of Napier's work on logarithms, archived from the original on 3 December 2002, retrieved 12 October 2010 Glaisher, James Whitbread Lee (1911), "Logarithm" , in Chisholm, Hugh (ed.), Encyclopædia Britannica, vol. 16 (11th ed.), Cambridge University Press, pp. 868–77 Retrieved from " Logarithmic functions are referred to as the inverse of the exponential function. In other words, the functions of the form f(x) = logbx are called logarithmic functions where b represents the base of the logarithm and b > 0. Concept of logarithm in mathematics is used for changing multiplication and division problems to problems of addition and subtraction.Logarithmic functions can be easily converted into exponential functions and vice versa. The formula for converting the logarithmic functions to exponential function is given by: ax = p ⇔ x = logapExample of Logarithmic FunctionsSome examples of the logarithmic functions are listed below:y(x) = log3 xp(y) = log (y + 6) - 5z(x) = 5ln x Common Logarithmic FunctionLogarithmic function which contains the logarithm of base 10 is called common logarithmic function. The common logarithm is represented as the log10 or log. The common logarithmic function is of the formf(x) = log10xNatural Logarithmic FunctionLogarithmic function which contains the logarithm of base e is called common logarithmic function. The common logarithm is represented as the loge or ln. The natural logarithmic function is of the form f(x) = logexDomain and Range of Logarithmic FunctionsBelow we will discuss about the domain and range of the logarithmic functions.Domain of Logarithmic FunctionsDomain of the fundamental logarithmic function i.e., y = log x is all the positive real numbers since the logarithmic function is defined for the positive numbers only i.e., x > 0. To find the domain of the other logarithmic functions put the term with log > 0 and find the value of variable. Domain of the given logarithmic function is given by (value of variable, ∞).Domain of log x = All Positive Real NumbersorDomain of log x = (0, ∞)Range of Logarithmic FunctionsRange of the logarithmic function is defined by putting the different values of x in the given logarithmic functions. The range of the logarithmic function is set of all real numbers. Range of Logarithmic function = R (Real Numbers)In summary:Domain of log function y = log x is x > 0 (or) (0, ∞)Range of any log function is the set of all real numbers (R)Logarithmic Function GraphWe know that, logarithmic functions are the inverse of the exponential functions. So, the graph of both the functions are symmetrical about line y = x. Also, the domain of the logarithmic function log x is set of all the positive real numbers and the range is the set of all real numbers. Logarithmic graph is plotted with the help of domain and range of the logarithmic function. We find the x- intercept of the logarithmic function and plot the logarithmic graph. The y-intercept of the logarithmic graph is not defined. Graph of both logarithmic function and exponential function is added below:Properties of Logarithmic FunctionThe properties of the logarithmic functions help us to solve the logarithmic functions. The several properties of logarithmic functions are listed below:logb1 = 0logb b = 1logb (pq) = logb p + logb qlogb (p/q) = logb p - logb q logb px = x logb plogb p = (logc p) / (logc b)Derivative of Logarithmic FunctionThe derivative of logarithmic function logex is 1/x. The derivative of the logarithmic function with base 'a' i.e., logax is 1 / (x ln a). The formula for derivatives of logarithmic function is given below.Derivative of ln x, i.e. (d/dx) (loge x) = 1/xDerivative of log x, i.e. (d/dx) (logax) = 1 / (x logea)Integral of Logarithmic FunctionIntegral of logarithmic function is calculated using the ILATE rule. The value of the integral of the logarithmic functions given below. ∫logex dx = x (logex - 1) + C∫log x dx = x (log x - 1) + C Solved Examples on Logarithmic FunctionsExample 1: Evaluate log 20 - log 2Solution:Let y = log 20 - log 2Using formula: logb (p/q) = logb p - logb q y = log (20/ 2)y = log 10Using formula: logb b = 1y = 1Example 2: Solve: log927 + 5Solution:Let y = log927 + 5Using formula: logb p = (logc p) / (logc b)log927 = (log3 27) / (log3 9)log927 = (log3 33) / (log332)Using the formula: logb px = x logb plog927 = 3(log3 3) / 2(log33)Using formula: logb b = 1log927 = 3/ 2Putting above value in y.y = (3/2) + 5y = 13/5Example 3: Find the value of x when log2x + log2(x + 6) = 4.Solution: log2x + log2(x + 6) = 4Using formula: logb (pq) = logb p + logb qlog2 [x (x + 6)] = 4Using formula: ax = p ⇔ x = logap [x (x + 6)] = 24 [x (x + 6)] = 16x2 + 6x - 16 = 0x = 2 or - 8Example 4: Find the domain and range of given logarithmic function y = log (6x - 24) + 7.Solution:y = log (6x - 24) + 7To find the domain of the given function put 6x - 24 > 06x - 24 > 06x > 24x > 4Domain of the given logarithmic function = (4, ∞)We know that,Range of any logarithmic function is set of all real numbers.